13

DISPVTATIO MATHEMATICA

DE

# SVBSECANTIBVS LINEARVM SECVNDI ORDINIS.

QVAM

*RECTORE*

VNIVERSITATIS EBERHARDINAE CAROLINAE

*MAGNIFICENTISSIMO*

*SERENISSIMO atque POTENTISSIMO DVCE ac DOMINO*

DOMINO

CAROLO,

DVCE WIRTEMBERGIÆ ET TECCIÆ REGNANTE

rel. rel.

CONSENTIENTE AMPLISSIMO PHILOSOPHORVM ORDINE

*PRAESIDE*

*VIRO EXCELLENTISSIMO atque DOCTISSIMO*

DOMINO

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ACADEMIAE SCIENTIARVM REGIAE BORVSSICAE SODALI,

PRAECEPTORE AC PATRONO SVO PIE DEVENERANDO

PRO GRADV MAGISTRI SIVE DOCTORIS PHILOSOPHIAE

RITE CONSEQVENDO

*DIE AVGVSTI ANNI MDCCLXXIX.*

PVBLICE DEFENDET

AVCTOR

FRIDERICVS GVILIELMVS KOESTLIN, *Brackenhemienſis*,

SERENISSIMI STIPENDIARIVS ET MAGISTERII PHILOS. CANDIDATVS.

*TVBINGÆ*, LITERIS SIGMVNDIANIS.

## §. I.

Subiit me aliquando, generationem variarum conoidum ſphæroidumque animo penſitantem, cogitatio, nonne, ſi tales in illis applicarentur ſectiones, quales in cono regulari, novæ ea ratione prodirent determinarique poſſent lineæ curvæ? Sectiones omnes poſſibiles in corporibus, ex terra lutoſa in hunc finem a me confectis, inſtitui, ſed quid harum ſola contemplatione confici potuit? Cum igitur extrema harum ſectionum puncta geometrice determinare aggreſſus eſſem, graviter offendi in linea extra curvam generantem indeterminate ab axe aut diametro aliqua per prolongatam chordam ſive ſecantem defecta. Satis diu hoc in problemate hærens recordatus ſum animadverſionis, quam in lectione Celeberrimi *de la* CHAPELLE in methodum ejus univerſalem *a*) determinandi rationem tangentium curvarum linearum ad quantitates cognitas, feceram, ex qua mihi lux illico oriebatur.

## §. II.

Etenim recte ille quidem conſiderat tangentes uti ſecantes, quorum puncta interſectionis in uno curvæ puncto confluunt, ſed in



*a*) In libro ipſius (*Abhandlung von den Kegelſchnitten*) e gallica lingua tranſpoſito, pag. 82, 83, 278, 400, 401.

in applicatione hujus theſeos non parum offendit. Putat ſecantis *TMD* fig. I. puncta interſectionis *M* & *D*, ſi *TMD* in *T* quaſi centro moveatur verſus ſiniſtram, ita confluere, ut *D*, quod movendo magis magisque accedit ad *M*, tandem cadat in *M*, adeoque, ducta ordinata *MP*, ſubtangens prodeat = *PT*. Sed fallit omnino; neque enim hac ratione confluent puncta, neque ſubtangens erit = *PT*, ſed potius illa in puncto aliquo intermedio *N* confluent; hæc autem erit = *TX*. Primum enim, ſi movetur *TMD* circa *T*, omnia ejus moventur puncta excepto centro *T*, adeoque etiam movetur punctum interſectionis *M*, & quidem appropinquabit verſus *D*, & *D* verſus *M*, usque dum in vertice *N* partis curvæ ſummo coincidant: deinde, ſi in *M* eſſet confluxus, ſequeretur, quod punctum *M* eſſet locus, ubi & tangens a *T* ducta tangat, & ſecans ab eodem puncto *T* ducta ſecet curvam, adeoque *TM* & tangentis & ſecantis pars ſimul eſſet, quod omnino eſt impoſſibile. Idem error in Ellipſi & Hyperbola pag. 278, 400, 401. manifeſto eſt commiſſus.

## §. III.

Aſſumendum mihi potius hic videbatur centrum in puncto *M*, *b*) in quo linea ſecans *TMD* a *T* verſus dextram *A* ſe vertat, usque dum ceſſet ſecare curvam, hoc eſt, donec in tangentem tranſeat, quæ in puncto *M* erit applicata. Patebit cadere movendo *D* in *M*, eodemque tempore quieſcere *T* in *G*, adeoque, ducta ordinata *MP*, prodire ſubtangentem = *GP*. Sit *PB* = *d*; *AP* = *x*; *PT* = *ſ*; *TG* = *ξ*; Calculus hoc modo procedet:

BD²

*b*) Æquipolleret eademque veritas prodiret, ſi centrum aſſumeretur aut in *T*, aut in *M*, aut in *D*; ſed *M* hic magis ſe nobis præſtabit idoneum.

$$BD^2 : PM^2 = x + d : x$$
$$BD^2 : PM^2 = (ſ + d)^2 : ſ^2$$ adeoque
$$x + d : x = (ſ + d)^2 : ſ^2$$ ſive
$$d : x = 2ſd + d^2 : ſ^2$$ Ergo
$$dſ^2 = 2ſdx + d^2x$$
$$ſ^2 = 2ſx + dx;\ R)$$

Eo ipſo autem momento, quo *TMD* fiet tangens, evaneſcet $d$, adeoque $d = 0$, & ſimul erit $ſ = ſ - \xi$. In æquatione *R* igitur notatis hisce mutationibus, mutando $ſ$ in $ſ - \xi$ delendoque id, quod ductum eſt in $d = 0$; erit

$$(ſ - \xi)^2 = 2x . \overline{ſ - \xi}$$ adeoque
$$ſ - \xi = 2x.$$

§. IV.

Et hic ſponte ſe mihi obtulit linea illa indeterminate in *T* a ſecante *TMD* defecta, in cujus determinatione hæc noſtra verſetur diſputatio.

§. V.

Sine demonſtratione patet, tangentem *GMD* in ſuperiore puncto *M* interſectionis a ſecante factæ minorem abſcindere axis prolongati partem, quam ſecans *TMD* abſcindit; majorem autem aſcindere tangentem *FD*, quæ ad inferius interſectionis punctum *D* ducta eſt. Hæ autem portiones axis prolongati, quas tangentes atque ordinatæ, ab eodem curvæ puncto ductæ, intercipiunt, vocantur ſubtangentes. Eodem igitur jure, quo lineæ *FB* & *GP* ſubtangentes nuncupantur, nuncupem lineas *TP* & *TB* SVBSECANTES & quidem *TB* ſubſecantem majorem, *TP* autem minorem, & ſic etiam lineam *TMD* ſecantem majorem, *TM* autem minorem,

 quan-

quamquam quidem *TM* proprie nuncupari ſecans non poteſt, tamen habet directionem ad ſecantem & majoris pars eſt: aut ut brevitati ſtudeamus, ſit Subſecans Subtangens

| | |
|---|---|
| Subſecans | Subtangens |
| Major = ſubſ. $\alpha$ | Maj. = ſubt. $\alpha$ |
| Minor = ſubſ. $\beta$ | Min. = ſubt. $\beta$ |
| Secans | Tangens |
| Maj. = ſec. $\alpha$ | Maj. = tang. $\alpha$. |
| Min. = ſec. $\beta$ | Min. = tang. $\beta$. |

## VI.

Faciamus itaque periculum, nonne determinari poſſit harum ſubſecantium ratio ad quantitatem cognitam? Sit $DB = z$; $MP = y$; $AP = x$; $AB = w$; $TG = \xi$; Parameter axis $= p$.

$y$ ſemper minor erit, quam $z$; $z$ itaque conſiderari poteſt, uti unitas, cujus partes quasdam continet $y$, ſive $y$ ſemper = fractioni τȣ $z$, quam exprimam per $\frac{n}{m}z$; $n$ ſcilicet $<$ $m$ aſſumto: $m$ & $n$ etiam exprimant numeros irrationales; in genere qualemcunque rationem τȣ $y$ ad τo $z$. Conſtat eſſe $z^2 = wp$ & $y^2 = px$; ergo $px = \frac{n^2}{m^2}z^2 = \frac{n^2}{m^2}wp$, adeoque $x = \frac{n^2}{m^2}w$. Hoc eſt $x$ ſemper = fractioni τȣ $z$, $y$ exprimenti, in ſe ductæ & multiplicatæ cum abſciſſa majoris ordinatæ; ſive, ſi minor ordinata eſt pars $r^{ta}$ majoris ordinatæ, tum minor abſciſſa eſt pars $r^{2ta}$ majoris abſciſſæ. Determinata itaque ratione τȣ $x$ ad $w$, habemus ſimul rationem ſubtangentis ad τo $w$, quæ eſt $= 2x$ vid. §. VI. adeoque $= \frac{2n^2}{m^2}w$. Conſideremus nunc triangula ſimilia *TMP* & *TDB* erit

$z : y$

$z : y =$ ſubſ. $\alpha$ : ſubſ. $\beta$. Eſt autem $y = \frac{n}{m}z$; ergo etiam ſubſ. $\beta = \frac{n}{m}$ ſubſ. $\alpha$;

$$\text{ſubſ. } \alpha = w + \frac{n^2}{m^2}w + \xi = \frac{m^2 + n^2}{m^2}w + \xi \quad \text{A)}$$

$$\frac{n}{m}\text{ ſubſ. } \alpha = \frac{2n^2}{m^2}w + \xi;$$

$$n\text{ ſubſ. } \alpha = \frac{2n^2}{m}w + m\xi$$

$$\text{ſubſ. } \alpha = \frac{2n}{m}w + \frac{m}{n}\xi \quad \text{B)}$$

Subſtituto valore pro ſubſ. $\alpha$ ex æquatione $A$ in $B$ eſt

$$\frac{m^2 + n^2}{m^2}w + \xi = \frac{2n}{m}w + \frac{m}{n}\xi$$

$$\frac{m}{n}\xi - \xi = \frac{m^2 + n^2}{m^2}w - \frac{2n}{m}w \quad \text{ſive}$$

$$\frac{m - n}{n}\xi = \frac{(m - n)^2}{m^2}w$$

$$\xi = \frac{(m - n)n}{m^2}w.$$

Et ſic determinavimus partes conſtituentes ſubſ. $\beta$; nempe $\xi + 2x$. Adeoque

$$\text{ſubſ. } \beta = \frac{2n^2}{m^2}w + \frac{(m - n)n}{m^2}w = \frac{mn + n^2}{m^2}w$$

$$= \frac{(m + n)n}{m^2}w.$$

Subſ.

Subſ. $\alpha$ $= PB +$ ſubſ. $\beta$; $PB = w - x = \frac{m^2 - n^2}{m^2} w$;

Itaque ſubſ. $\alpha = \frac{m^2 - n^2}{m^2} w + \frac{(m+n)n}{m^2} w$

$$= \frac{m^2 + mn}{m^2} w = \frac{m+n}{m} w.$$

## §. VII.

### *Exemplum* I.

$y = \frac{n}{m} z$; $x \frac{n^2}{m^2} w$; $\xi = \frac{\overline{m-n}.n}{m^2} w$; ſubſ. $\beta = \frac{\overline{m+n}.n}{m^2} w$; ſubſ. $\alpha = \frac{m+n}{m} w$.

| Sit $y$ | & erit $x$ | $\xi$ | ſubſ. $\beta$ | ſubſ. $\alpha$ |
|---|---|---|---|---|
| = | = | = | = | = |
| $\frac{1}{2} z$; | $\frac{1}{4} w$; | $\frac{1}{4} w = x$; | $\frac{3}{4} w = 3x$; | $\frac{3}{2} w = 6x$; |
| $\frac{1}{3} z$; | $\frac{1}{9} w$; | $\frac{2}{9} w = 2x$; | $\frac{4}{9} w = 4x$; | $\frac{4}{3} w = 12x$; |
| $\frac{1}{4} z$; | $\frac{1}{16} w$; | $\frac{3}{16} w = 3x$; | $\frac{5}{16} w = 5x$; | $\frac{5}{4} w = 20x$; |
| $\frac{1}{5} z$; | $\frac{1}{25} w$; | $\frac{4}{25} w = 4x$; | $\frac{6}{25} w = 6x$; | $\frac{6}{5} w = 30x$; |
| $\frac{1}{6} z$; | $\frac{1}{36} w$; | $\frac{5}{36} w = 5x$; | $\frac{7}{36} w = 7x$; | $\frac{7}{6} w = 42x$; |
| $\frac{1}{m} z$; | $\frac{1}{m^2} w$; | $(m-1)x$; | $(m+1)x$; | $m(m+1)x$; |
| $\frac{1}{\infty} z$; | $\frac{1}{\infty^2} w$; | $(\infty - 1)x$; | $(\infty + 1)x$; | $(\infty^2 + \infty)x$. |

*Exemplum*

### *Exemplum* II.

| Sit $y$ | & erit $x$ | $\xi$ | ſubſ. $\beta$ | ſubſ. $\alpha$ |
|---|---|---|---|---|
| = | = | = | = | = |
| $\frac{1}{2}z$; | $\frac{1}{4}w$; | $\frac{1}{4}w = x$; | $\frac{3}{4}w = 3x = (2+\frac{1}{1})x$; | $\frac{3}{2}w = 6x = (2+\frac{4}{1})x$. |
| $\frac{2}{3}z$; | $\frac{4}{9}w$; | $\frac{2}{9}w = \frac{x}{2}$; | $\frac{10}{9}w = \frac{5}{2}x = (2+\frac{1}{2})x$; | $\frac{5}{3}w = \frac{15x}{4} = (2+\frac{7}{4})x$. |
| $\frac{3}{4}z$; | $\frac{9}{16}w$; | $\frac{3}{16}w = \frac{x}{3}$; | $\frac{21}{16}w = \frac{7}{3}x = (2+\frac{1}{3})x$; | $\frac{7}{4}w = \frac{28x}{9} = (2+\frac{10}{9})x$. |
| $\frac{4}{5}z$; | $\frac{16}{25}w$; | $\frac{4}{25}w = \frac{x}{4}$; | $\frac{36}{25}w = \frac{9}{4}x = (2+\frac{1}{4})x$; | $\frac{9}{5}w = \frac{45x}{16} = (2+\frac{13}{16})x$. |
| $\frac{5}{6}z$; | $\frac{25}{36}w$; | $\frac{5}{36}w = \frac{x}{5}$; | $\frac{55}{36}w = \frac{11}{5}x = (2+\frac{1}{5})x$; | $\frac{11}{6}w = \frac{66x}{25} = (2+\frac{16}{25})x$. |
| $\frac{6}{7}z$; | $\frac{36}{49}w$; | $\frac{6}{49}w = \frac{x}{6}$; | $\frac{78}{49}w = \frac{13}{6}x = (2+\frac{1}{6})x$; | $\frac{13}{7}w = \frac{91}{36}w = (2+\frac{19}{36})x$. |
| $\frac{n}{n+1}z$; | $\frac{n^2}{(n+1)^2}$; | $\frac{x}{n}$; | $\left(2+\frac{1}{n}\right)x$; | $\left(2+\frac{3n+1}{n^2}\right)x$. |

### *Exemplum* III.

| Sit $y$ | & erit $x$ | $\xi$ | ſubſ. $\beta$ | ſubſ. $\alpha$ |
|---|---|---|---|---|
| = | = | = | = | = |
| $\frac{1}{\sqrt{2}}z$; | $\frac{1}{2}w$; | $\frac{\sqrt{2}-1}{2}w$; | $\frac{\sqrt{2}+1}{2}w$; | $\frac{\sqrt{2}+1}{\sqrt{2}} = \frac{\sqrt{2}+2}{2}w$. |
| $\frac{\sqrt{2}}{\sqrt{3}}z$; | $\frac{2}{3}w$; | $\frac{\sqrt{6}-2}{3}w$; | $\frac{\sqrt{6}+2}{3}w$; | $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}+3}{3}w$. |
| $\frac{\sqrt{3}}{2}z$; | $\frac{3}{4}w$; | $\frac{2\sqrt{3}-3}{4}w$; | $\frac{2\sqrt{3}+3}{4}w$; | $\frac{\sqrt{3}+2}{2} = \frac{3{,}73205\ldots}{2} = 1{,}8660254\ldots w$. |
| $\frac{\sqrt{r}}{\sqrt{(r+1)}}z$; | $\frac{r}{r+1}w$; | $\frac{\sqrt{(r^2+r)}-r}{r+1}w$; | $\frac{\sqrt{(r^2+r)}+r}{r+1}w$; | $\frac{\sqrt{(r+1)}+\sqrt{r}}{\sqrt{r+1}}w = \frac{\sqrt{(r^2+r)}+r+1}{r+1}w$. |

### *Exemplum* IV.

Sit $= \frac{n}{m} = \frac{\sqrt{128}}{\sqrt{200}}z = \frac{\sqrt{16}\,.\,\sqrt{8}}{\sqrt{25}\,.\,\sqrt{8}}z = \frac{4}{5}z$; reliqua vid. Ex. II.

Sit $= \frac{\sqrt{35}}{\sqrt{40}}z = \frac{\sqrt{7}\,.\,\sqrt{5}}{\sqrt{8}\,.\,\sqrt{5}}z = \frac{\sqrt{7}}{\sqrt{8}}z = \frac{\sqrt{56}}{8}z$; & erit

$$x = \tfrac{56}{64}w;\ \xi = \frac{(8-\sqrt{56})\sqrt{56}}{8\,.\,8}w = \frac{\sqrt{56}-7}{8}w$$
$$= \frac{2\sqrt{14}-7}{8}w$$

Subſ. $\beta = \frac{2\sqrt{14}+7}{8}w$; ſubſ. $\alpha = \frac{8+\sqrt{56}}{8}w$
$= \frac{4+\sqrt{14}}{4}w$.

Sit $= \frac{\sqrt[3]{63}}{\sqrt{63}}w = \frac{63^{\frac{1}{3}}\,.\,63^{\frac{1}{2}}}{63}w = 63^{-\frac{1}{6}}\,.\,w = \frac{1}{\sqrt[6]{63}}w$; & erit

$x = \frac{1}{63^{\frac{1}{3}}}w = \frac{1}{\sqrt[3]{63}}w$; $\xi = \frac{\sqrt[6]{63}-1}{\sqrt[6]{63}\,.\,\sqrt[6]{63}}w$; atque numeratore & denominatore ducto in $\sqrt[3]{63^2}$; $= \frac{\sqrt[6]{63^5}-\sqrt[3]{63^2}}{63}w$.

Subſ. $\beta = \left(\frac{\sqrt[6]{63^5}+\sqrt[3]{63^2}}{63}\right)w$; ſubſ. $\alpha = \left(\frac{\sqrt[6]{63}+1}{\sqrt[6]{63}}\right)w$
$= \left(1+\frac{1}{\sqrt[6]{63}}\right)w = \left(\frac{63+\sqrt[6]{63^5}}{63}\right)w$.

Not. Si $n = m$ aſſumimus, valorem ſubtangentis $= 2x$ invenimus. Nam ſi

$\frac{n}{m}z$

$\frac{n}{m}z = \frac{1}{1}z$; & erit $x = w$; $\xi = 0$; ſubſ. $\beta = 2x$; ſubſ. $\alpha = 2x$; hoc eſt ſubtangens $= 2x$.

## §. VIII.

$TA = \frac{m+n}{m}w - w = \frac{n}{m}w$. Ergo $TA : w = y . z$.

## §. IX.

Et ſecundum æquationem A) §. VI. eſt ſubſ. $\alpha - \xi = \frac{m^2 + n^2}{m^2}w = GB$.

## §. X.

Subt. $\alpha$. — Subt. . $\beta = FB - GP = 2w - \frac{2n^2}{m^2}w = \frac{(m^2 - n^2).2}{m^2}w = GF + PB$. Sed $AB = AF$; & $AP = AG$; adeoque $GF = PB$; hoc eſt $\frac{\frac{(m^2 - n^2)2}{m^2}w}{2} = \frac{m^2 - n^2}{m^2}w = GF = PB$.

## §. XI.

Subt. $\alpha$ — ſubſ. $\alpha = FT = \frac{m^2 - n^2}{m^2}w - \frac{\overline{m-n}.n}{m^2}w = \frac{m-n}{m}w$; Adeoque $FT . TB = \frac{m-n}{m}w \times \frac{m+n}{m}w = \frac{m^2 - n^2}{m^2}w^2$; Hinc $\frac{m^2 - n^2}{m^2}w^2 : \frac{m^2 - n^2}{m^2}w$ §. X. $= w^2 : w = w : 1 = BF^2 : 2BF =$ ſubt. $\alpha^2$ : 2ſubt. $\alpha$.

## §. XII.

$$\text{Sec. } \alpha^2 = \left(\frac{m+n}{m}w\right)^2 + z^2 = \frac{\overline{m+n}^2 \,.\, w^2 + m^2 p w}{m^2}$$

$$= \frac{pm^2 + w \,.\, \overline{m+n}^2}{m^2}w; \text{ adeoque}$$

$$\text{Sec. } \alpha = \frac{\sqrt{(pm^2 + w\,\overline{m+n}^2)}}{m}w.$$

## §. XIII.

$$\text{Sec. } \beta^2 = \left(\frac{n}{m}z\right)^2 + \left(\frac{m+n \,.\, n}{m^2}w\right)^2 = \frac{n^2 m^2 p w}{m^4}$$

$$+ \frac{\overline{m+n}^2\, n^2 w^2}{m^4} = \frac{(pm^2 + w \,.\, \overline{m+n}^2)n^2}{m^4}w; \text{ adeoque}$$

$$\text{Sec. } \beta = \frac{n\sqrt{(pm^2 + w \,.\, \overline{m+n}^2)}}{m^2}w.$$

## §. XIV.

Brevius commodiusque rationes ſubſecantium in tabulis hisce proponam. Sit $m - n = D$; $m + n = S$.

*Tabula* I.

| | | |
|---|---|---|
| Subſ. $\alpha$ : Subſ. $\beta$ | $= m : n$ | $= S+D : S-D$ |
| » : $\xi$ | $= m+n : \frac{\overline{m-n}\,.\,n}{m}$ | $= S : \frac{D(S-D)}{S+D}$ |
| » : $x$ | $= m+n : \frac{n^2}{m}$ | $= S : \frac{(S-D)^2}{2(S+D)}$ |
| » : $w$ | $= m+n : m$ | $= S : \frac{S+D}{2}$ |
| » $-w$ : $w$ | $= n : m$ | $= S-D : S+D$ |
| : ſubt. $\beta$ | $= m+n : \frac{2n^2}{m}$ | $= S : \frac{(S-D)^2}{S+D}$ |
| » : » $\alpha$ | $= m+n : 2m$ | $= S : S+D$ |
| » : $z^2$ | $= m+n : mp$ | $= S : \frac{(S+D)p}{2}$ |
| » : $y^2$ | $= m+n : \frac{n^2p}{m}$ | $= S : \frac{(S-D)^2p}{2(S+D)}$ |
| » : ſec. $\alpha^2$ | $= m+n : \frac{pm^2 + \overline{wm+n}^2}{m}$ | |
| » : » $\beta^2$ | $= m+n : \frac{(pm^2 + \overline{wm+n}^2).n^2}{m^3}$ | |
| : tang. $\alpha^2$ | $= m+n : m(4w+p)$ | |
| » : » $\beta^2$ | $= m+n : \frac{(4n^2w+pm^2)n^2}{m^3}$ | |
| » : $w - x$ | $= m : m-n$ | $= \frac{S+D}{2} : D$ |
| : ſubſ. $\alpha - w$ | $= m+n : n$ | $= S : \frac{S-D}{2}$ |
| » : ſubt. $\alpha$ —ſubſ. $\alpha$ | $= m+n : m-n$ | $= S : D.$ |



*Tabula*

*Tabula* II.

| | | | |
|---|---|---|---|
| Subſ. $\beta$ : $\xi$ | $= m+n : m-n$ | $= S : D$ |
| ,, : $x$ | $= m+n : n$ | $= S : \frac{S-D}{2}$ |
| ,, $-x$ : $x$ | $= m : n$ | $= S+D : S-D$ |
| ,, : ſubt. $\alpha$ | $= m+n : \frac{2m^2}{n}$ | $= S : \frac{(S+D)^2}{S-D}$ |
| ,, : ,, $\beta$ | $= m+n : 2n$ | $= S : S-D$ |
| ,, : $z^2$ | $= m+n : \frac{m^2p}{n}$ | $= S : \frac{(S+D)^2p}{2(S-D)}$ |
| ,, : $y^2$ | $= m+n : np$ | $= S : \frac{(S-D)p}{2}$ |
| ,, : ſec. $\alpha^2$ | $m+n : \frac{pm^2+w\,.\,\overline{m+n}^2}{n}$ | |
| ,, : ,, $\beta^2$ | $= m+n : \frac{(pm^2+w\,\overline{m+n}^2)n}{m^2}$ | |
| ,, : tang. $\alpha^2$ | $= m+n : \frac{(4w+p)m^2}{n}$ | |
| ,, : ,, $\beta^2$ | $= m+n : \frac{(pm^2+4n^2w)n}{m^2}$ | |
| ,, : $w-x$ | $= n : m-n$ | $= \frac{S-D}{2} : D$ |
| ,, : ſubſ. $\alpha - w$ | $= m+n : m$ | $= S : \frac{S+D}{2}$ |
| ,, : ſubt. $\alpha$ —ſubſ. $\alpha$ | $= m+n : \frac{m\,.\,\overline{m-n}}{n}$ | $= S : \frac{(S+D)D}{S-D}$ |

§. XV.

§. XV.

Ex quibus iterum fluunt rationes aliæ:
Collatis ſcilicet rationibus 1. & 5. Tab. I. & 3. Tab. II.; habebit ſe ſubſ. $\beta$ : ſubſ. $\alpha$ = ſubſ. $\alpha - w$ : $w = x$ : ſubſ. $\beta - x$; ſive

$$= AT : w = x : AT;$$ adeoque $AT^2 = wx$.

Et coll. ultima rat. Tab. I. & 1. Tab. II. erit

ſubſ. $\beta$ : ſubſ. $\alpha = \xi : FT$.

§. XVI.

Quod ſubſecantes diametrorum attinet, æquipollet, an ſubſecantem ad ordinatam majorem $bd$, an ad ordinatam $b\delta$ quæramus. Etenim ponamus prodire pro alterutra aut majorem aut minorem, falſitas ſi elucebit: ſi ſecans $\vartheta\mu\delta$ abſcinderet

Imo majorem diametri portionem, quam ſec. $dm\vartheta$; eſſet

$$\mu p : \delta b = p\vartheta + v : b\vartheta + v \quad \&$$

$$mp : db = p\vartheta : b\vartheta; \text{ ſed } \frac{mp}{db} = \frac{\mu p}{\delta b}$$

adeoque etiam $\frac{p\vartheta}{b\vartheta} = \frac{p\vartheta + v}{b\vartheta + v}$ eſſe deberet, quod manifeſto eſt falſum.

IIdo minorem abſcinderet; eſſet

$$\mu p : \delta b = p\vartheta - v : b\vartheta - v \quad \&$$

$$mp : db = p\vartheta : b\vartheta$$ adeoque

$\frac{p\vartheta}{b\vartheta} = \frac{p\vartheta - v}{b\vartheta - v}$ foret, quod iterum eſt abſurdum. Ergo ſi nec majorem, nec minorem alterutra poteſt abſcindere diametri portionem, vergere debent in uno eodemque diametri puncto, & dare pro utraque ordinata ſubſecantem unam eandemque.

§. XVII.

## §. XVII.

Facile, credo, mihi nunc ignoſcetur, ſi de ſubſecantibus diametrorum demonſtrationem non exhibeam; quisque enim facile perſpiciet, demonſtrationem hanc ob analogiam cum illa §. VI. plane eſſe ſupervacaneam. Formulæ itaque ac rationes propoſitæ quadrant etiam in determinandis functionibus diametrorum ad abſciſſas ipſarum majores, ſi formulas ſecantium tangentiumque excipias, quia ibi de coordinatis rectangulis, hic autem de ſcalenis eſt ſermo. Pro ſecantibus igitur quæramus formulas.

Abſciſſa $ab$ ſit $= \mathfrak{w}$; parameter $= \pi$; ſec. $\vartheta\mu\delta =$ 1ma ſec. $\alpha$; $\vartheta\mu =$ 1ma ſec. $\beta$; $\vartheta A d =$ 2da ſec. $\alpha$; $\vartheta A =$ 2da ſec. $\beta$.

1ma ſec. $\alpha^2 = (\text{ſubſ. } \alpha^2 = \vartheta b^2) + b\delta^2 - 2\vartheta b \,.\, b\delta \,.\, (-\text{coſ. } \vartheta b\delta)$ c)

$$= \left(\frac{m+n}{m}\mathfrak{w}\right)^2 + \mathfrak{w}\pi - 2\left(\frac{m+n}{m}\mathfrak{w}\right)\surd(\mathfrak{w}\pi) \,.\, (-\text{coſ. } \vartheta b\delta)$$

$$= \frac{\overline{m+n}^2 \,.\, \mathfrak{w}^2 + m^2\mathfrak{w}\pi + 2m\,\overline{m+n}\,\mathfrak{w}\surd(\mathfrak{w}\pi) \,.\, \text{coſ. } \vartheta b\delta}{m^2}$$

$$= \frac{\mathfrak{w}\pi \times m^2 + \mathfrak{w}\surd(\mathfrak{w}\pi)\,\text{coſ. } \vartheta b\delta \times 2m\,\overline{m+n} + \mathfrak{w}^2 \times \overline{m+n}^2}{m^2}$$

Itaque 1ma ſec. $\alpha$.

$$= \frac{\surd[\mathfrak{w}\pi \times m^2 + \mathfrak{w}\surd(\mathfrak{w}\pi)\,\text{coſ. } \vartheta b\delta \times 2m\,\overline{m+n} + \mathfrak{w}^2 \times \overline{m+n}^2]}{m}$$

2da ſec. $\alpha$

$$= \frac{\surd[\mathfrak{w}\pi \times m^2 - \mathfrak{w}\surd(\mathfrak{w}\pi)\,\text{coſ. } \vartheta b\delta \times 2m\,\overline{m+n} + \mathfrak{w}^2 \times \overline{m+n}^2]}{m}$$

Adeo-

c) vid. Celeberr. Kæstner. Trigonom. 20. Saz. nr. 33, 1. Zuſaz.

Adeoque 1. ſec. $\beta = \frac{n\sqrt{(\,.\,.\,m^2 + .\,.\,2m\overline{m+n} + .\,\overline{m+n}^2)}}{m^2}$

2. ſec. $\beta = \frac{n\sqrt{(\,.\,.\,m^2 - .\,.\,2m\overline{m+n} + .\,\overline{m+n}^2)}}{m^2}$

Ergo ſubſ. $\alpha$ ; ſec. $\alpha^2 = m + n : \frac{\pi m^2 \pm \sqrt{(\mathfrak{w}\pi)}\,\text{coſ.}\,\ldots + \mathfrak{w}\,.\,\overline{m+n}^2}{m}$

„ : „ $\beta^2 = m + n ; \frac{(\pi m^2 \pm \sqrt{\mathfrak{w}\pi}\,.\,\text{coſ.}\ldots + \mathfrak{w}\,\overline{m+n}^2)n^2}{m^3}$

ſubſ. $\beta$ : „ $\alpha^2 = m + n ; \frac{\pi m^2 \pm \sqrt{\mathfrak{w}\pi}\,.\,\text{coſ.}\,.\,.\, + \mathfrak{w}\,\overline{m+n}^2}{n}$

„ : „ $\beta^2 = m + n : \frac{(\pi m^2 \pm \sqrt{\mathfrak{w}\pi}\,.\,\text{coſ.}\,.\,.\, + \mathfrak{w}\overline{m+n}^2)n}{m^2}$.

## §. XVIII.

Problema.

Reducere functiones diametrorum ad functiones axis.

Solutio.

Data ubicunque ſit in parabolæ curva diameter, ſitque ordinata quæcunque ad illam; tamen ſemper poterit ordinatæ huic duci parallela ex puncto *A*, fig. II. vertice parabolæ, quæ ducta exhibet ordinatam ad eandem diametrum, quamque ordinatam dirigentem voco, omnesque aliæ ordinatæ cadent, aut infra, aut ſupra *A*. Conſideremus

Imo ſi cadit ordinata *infra A*.

Sit $ap = \mathfrak{x}$; $Ap = \mathfrak{y}$; $db = \mathfrak{z}$; parameter diam. $= \pi$. ducta tangente $\mu g C$ ad $\mu$ erit $\mu p : \mu A = gp : CA$; $\mu p = \frac{\mu A}{2}$; ergo $gp = \frac{CA}{2}$; $CA = x$; ergo $gp = 2\mathfrak{x} = \frac{x}{2}$; $\mathfrak{x} = \frac{x}{4}$; Porro $\mu p : \mu A = pV : AP$; Ergo $PV = \frac{AP}{2} = \frac{x}{2} = 2\mathfrak{x}$;

$ga = ap = \mathfrak{x} = \frac{x}{4} = \frac{n^2}{4m^2}w$; Porro

$A\mu^2 = x^2 + y^2 = \frac{n^4}{m^4}w^2 + \frac{n^2 w p}{m^2} = \frac{(wn^2 + pm^2)n^2 . w}{m^4}$;

Itaque $\mu p^2 = Ap^2 = \mathfrak{y}^2 = \frac{(wn^2 + pm^2)n^2 . w}{4m^4}$; Ergo

$\mathfrak{y}^2 : \mathfrak{x} = \frac{wn^2 + pm^2}{m^2} : 1 = \pi : 1$; nam $\mathfrak{x} : \mathfrak{y} = \mathfrak{y} : \pi$;

adeoque $\pi = \frac{\mathfrak{y}^2}{\mathfrak{x}} = \frac{(wn^2 + pm^2)\, n^2 w}{4m^4} \times \frac{4m^2}{n^2 w} = \frac{n^2 w + m^2 p}{m^2}$. d)

Porro ord. $\beta^2$ : ſec. $\beta^2$ = ord. $\alpha^2$ : ſec. $\alpha^2$; ſive

$$p\mu^2 . \left(\frac{\mu R}{2}\right)^2 = b\delta^2 : \left(\delta R - \frac{\mu R}{2}\right)^2$$

quæ collatis §§ XII & XIII. ita exprimuntur:

$mp^2$

d) Sed $\pi = 4x + p$; vid. DE LA CHAPELLE pag. 129, 130. adeoque $= \frac{4wn^2 + pm^2}{m^2}$? Omnino: Ibi vero ordin. $\mu p$ ad punctum $a$, ubi diameter curvam intrat, hic autem ad illud, in quo ordin. $A\mu$ cum curva vergit, eſt ducta.

$$\frac{(pm^2+wn^2)n^2w}{4m^4} : \frac{(pm^2+w\overline{m+n}^2)n^2w}{4m^4} = b\delta^2 : \frac{(pm^2+w\overline{m+n}^2)w}{m^2}$$

$$-\ \frac{2\cdot\sqrt{(pm^2+w\overline{m+n}^2)w}}{m} \times \frac{n\sqrt{(pm^2+w\overline{m+n}^2)w}}{2m^2}$$

$$+\ \frac{(pm^2+w\overline{m+n}^2)n^2w}{4m^4}$$

$$= \frac{(4m^2+n^2).(pm^2+w\overline{m+n}^2)w - 4nm\sqrt{(pm^2+..)}.(\sqrt{..m^2..})}{4m^4}$$

$$= \frac{(4m^2+n^2)(pm^2+w\overline{m+n}^2)w - 4nm(pm^2+w.\overline{m+n}^2)w}{4m^4}$$

$$= \frac{(4m^2+n^2-4nm)(pm^2+w\overline{m+n}^2)w}{4m^4}$$

$$= \frac{\overline{2m-n}^2(pm^2+w.\overline{m+n}^2)w}{4m^4} = \delta\vartheta^2 =$$ ſec. $\alpha$ diam.

Et $\delta\vartheta = \frac{\sqrt{(\overline{2m-n}^2(pm^2+w\overline{m+n}^2)w}}{4m^4} = \frac{\overline{2m-n}\sqrt{(pm^2+w\overline{m+n}^2)w}}{2m^2}$

Adeoque $b\delta^2 = \frac{p\mu^2\times\delta\vartheta^2}{\mu\vartheta^2} =$

$$\frac{(pm^2+wn^2)n^2w}{4m^4} \times \frac{\overline{2m-n}^2(pm^2+w.\overline{m+n}^2)w}{4m^4} \times \frac{4m^4}{(pm^2+w\overline{m+n}^2)n^2w}$$

$$= \frac{(pm^2+wn^2)w\,\overline{2m-n}^2}{4m^4}$$



$b\delta =$

$$b\delta = \frac{\overline{2m-n}\sqrt{(pm^2+wn^2)w}}{2m^2}.$$

Porro eſt $b\delta^2 = ab \,.\, \pi$ ; $\pi = \frac{pm^2+wn^2}{m^2}$ Ergo

$$ab = \frac{\overline{2m-n}^{\,2}(pm^2+wn^2)w}{4m^4} : \frac{pm^2+wn^2}{m^2} = \frac{\overline{2m-n}^{\,2}}{4m^2}w$$

adeoque $\mathfrak{z}^2 : \mathfrak{w} = \frac{\overline{2m-n}^{\,2}(pm^2+wn^2)w}{4m^4} : \frac{(2m-n)^2 w}{4m^2} = \pi : 1.$

Subſ. $\beta = \vartheta p = \frac{RA}{2} = \frac{n}{2m}w$

Subſ. $\beta - \mathfrak{y} = \vartheta a = \left(\frac{n}{2m} - \frac{n^2}{4m^2}\right)w = \frac{\overline{2m-n}\,n}{4m^2}w.$

Subſ. $\alpha = \vartheta b = \vartheta a + ab = \left(\frac{(2m-n)n}{4m^2} + \frac{(2m-n)^2}{4m^2}\right)w$

$$= \frac{(2m-n+n)\,\overline{2m-n}}{4m^2}w = \frac{2m-n}{2m}w$$

Subt. $\alpha = gb = 2\mathfrak{w} = \frac{\overline{2m-n}^{\,2}}{2m^2}w$

Subt. $\beta = gp = 2\mathfrak{y} = \frac{n^2}{2m^2}w$

Subſ. $\beta$ — ſubt. $\beta = \vartheta g = \frac{\xi}{2} = \zeta = \frac{\overline{m-n}\,n}{2m^2}w$

tang. $\beta^2$ diam. $= \mu g^2 = \frac{1}{4}$ tang. $\beta^2$ axis $= \frac{\left(\frac{4n^4}{m^4} + \frac{pn^2}{m^2}\right)w}{4}$

$$= \frac{(4wn^2+pm^2)n^2w}{4m^4}$$

tang.

tang. $\alpha^2$ diam. = $\delta f^2$ determinatur ſequenti modo:

$\delta R^2 : \delta \vartheta^2 = \delta Q^2 : \delta f^2$; ſive in expreſſionibus analyt.

$$\frac{(pm^2 + w\,\overline{m+n}^2)w}{m^2} : \frac{\overline{2m-n}^2\,(pm^2 + w\,.\,\overline{m+n}^2)w}{4m^4} = \overline{4w+pw} :$$

$$\frac{\overline{4w+pw}\,.\,\overline{2m-n}^2\,(pm^2 + w\,\overline{m+n}^2)w}{4m^4} \times \frac{m^2}{(pm^2 + w\,\overline{m+n}^2)w}$$

$$= \frac{\overline{4w+p}\;\overline{2m-n}^2\,w}{4m^2};$$ Ergo

tang. $\alpha . = \frac{\overline{2m-n}\sqrt{\overline{4w+pw}}}{2m}$.

Rationes propoſitas iterum ponamus in Tabulam.

*Tabula* III.

Rationes functionum diametrorum inter ſe, & quidem, ſi ordinata *infra* verticem axis $A$ cadunt.

| | | | |
|---|---|---|---|
| 𝔭 | : 𝔴 | = $n$ | : $\frac{\overline{2m-n}^2}{n}$ |
| 𝔶 | : 𝔷 | = $n$ | : $2m - n$ |
| 𝔭 | : 𝔶$^2$ | = $m$ | : $\frac{pm^2 + wn^2}{m}$ |
| 𝔴 | : 𝔷$^2$ | = „ | „ |
| 𝔴 | : ſubſ. $\beta$ | = $\frac{(2m-n)^2}{2m}$ | : $n$ |
| „ | : „ $\alpha$ | = $\frac{2m-n}{2}$ | : $m$ |

| | | | |
|---|---|---|---|
| $w$ | : ſubt. $\beta$ | $=\frac{(2m-n)^2}{2n}$ | : $n$ |
| „ | : „ $\alpha$ | $=1$ | : $2$ |
| „ | : $\zeta$ | $=\frac{(2m-n)^2}{2n}$ | : $m-n$ |
| tang. $\alpha^2$ | : tang. $\beta^2$ | $=(2m-n)^2$ | : $\frac{(4wn^2+pm^2)n^2}{(4w+p)m^2}$ |
| „ | : ſubt $\alpha$ | $=4w+p$ | : $2$ |
| „ | : „ $\beta$ | $=\frac{\overline{4w+p}\;\overline{2m-n}^2}{2n}$ | : $n$ |
| tang. $\beta^2$ | : ſubt. $\alpha$ | $=\frac{(4wn^2+pm^2)n^2}{2m^2}$ | : $(2m-n)^2$ |
| „ | : ſubt. $\beta$ | $=\frac{(4wn^2+pm^2)n^2}{2m}$ | : $m$ |
| ſubt. $\alpha$ | : ſubt. $\beta$ | $=\frac{(2m-n)^2}{n}$ | : $n$ |
| „ | : ſubſ. $\beta$ | $=\frac{(2m-n)^2}{m}$ | : $n$ |
| „ | : ſubſ. $\alpha$ | $=2m-n$ | : $m$ |
| ſubſ. $\alpha$ | : ſubſ. $\beta$ | $=2m-n$ | : $n$ |
| ſec. $\alpha$ | : ſec. $\beta$ | $=2m-n$ | : $n$ |
| „ | : ſubſ. $\alpha$ | $=\frac{\sqrt{(pm^2+w\,\overline{m+n}^2)}}{w}$ | : $m$ |
| „ | : „ $\beta$ | $=\frac{(2m-n)\sqrt{(pm^2+w\,\overline{m+n}^2)}}{mw}$ | : $n$ |

ſec.

$$\text{ſec. } \alpha : \text{tang. } \alpha = \frac{\sqrt{(pm^2+w\,\overline{m+n}^2)w}}{\sqrt{(4w+p)w}} : m$$

$$„ : „\ \beta = \frac{(2m-n)\sqrt{(pm^2+w\,\overline{m+n}^2)w}}{\sqrt{(4w^2+pm^2)}} : n$$

$$„\ \beta : \text{ſubſ. } \alpha = \frac{n\sqrt{(pm^2+w\,\overline{m+n}^2)w}}{(2m-n)w} : m$$

$$„ : „\ \beta = \frac{\sqrt{(pm^2+w\,\overline{m+n}^2)w}}{w} : m$$

$$„ : \text{tang. } \alpha = \frac{n\sqrt{(pm^2+w\,\overline{m+n}^2)w}}{(2m-n)\sqrt{(4w+p)w}} : m$$

$$„ : „\ \beta = \sqrt{(pm^2+w\,\overline{m+n}^2)w} : \sqrt{(4wn^2+pm^2)w}$$

$$\text{ſubſ. } \alpha : \text{tang. } \alpha = w : \sqrt{(4w+p)w}$$

$$„ : „\ \beta = 2m-n : \frac{n\sqrt{(4wn^2+pm^2)}}{mw}$$

$$\text{ſubſ. } \beta : \text{tang. } \alpha = n : \frac{(2m-n)\sqrt{(4w+p)w}}{w}$$

$$„ : „\ \beta = m : \frac{\sqrt{(4wn^2+pm^2)w}}{w}.$$

IIdo ſi ordinata *ſupra* ſecantem ſeu ordinatam dirigentem cadit, aut quod idem eſt, ſi ordinata ex puncto curvæ inter axem & diametrum eſt ducta.

Hic major abſciſſa eſt $= AP = w$; minor $AE = x$; major ord. $P\mu = z$; minor $EO = y$, & ſic etiam $p\mu = \mathfrak{z}$; $no = \mathfrak{y}$;

*ap*

$ap = \mathfrak{w}$; $an = \mathfrak{y}$; $T\mu =$ ſec. $\alpha$ axis; $To =$ ſec. $\beta$ axis; $\mu ſ =$ ſec. $\alpha$ diam.; $oſ =$ ſec. $\beta$ diam.

Nunc erit $A\mu^2 = w^2 + z^2 = (w + p)w$

$$p\mu^2 = \frac{(w+p)w}{4}$$

$ap = \frac{1}{4}w$; adeoque $\pi = w + p$; $\epsilon$) nam $\frac{p\mu^2}{ap} = \pi$;

ſubſ. $\alpha = ſp = \frac{1}{2}TA = \frac{n}{2m}w$;

$$ſa = \frac{n}{2m}w - ap = \left(\frac{n}{2m} - \frac{1}{4}\right)w = \frac{2n - m}{4m}w;$$

Porro $T\mu^2 : ſ\mu^2 = 1 : \frac{1}{4}$; adeoque $\mu ſ^2 =$ ſec. $\alpha^2 = T\mu^2 =$ $\frac{(pm^2 + \overline{wm + n}^2)}{4m^2}w$; vid. §. XII. $\mu ſ = \frac{\sqrt{(pm^2 + \overline{wm+n}^2)w}}{2m}$.

Vt nunc expreſſio pro ordinata $on$ prodeat, conſideremus triangula ſimilia $ſon$, $ſ\mu p$; erit

$ſ\mu^2$

$\epsilon$) Brevioribus gauderemus expreſſionibus pro caſu I. & generatim calculo reductionis functionum generaliori breviorique, ſi parametri $\pi$ haberemus pro utroque caſu I. & II. unam eandemque formulam $= w + p$; quod fieri poſſet, ſi uterque caſus ad $AP$ reduceretur; ſed ex alia parte magis compoſitis uti deberemus formulis, quæ tamen hac hucusque a me adhibita ratione ſunt ſimpliciſſima: exempli gratia, ſi ad $x$ omnia reducerentur, eſſet ſubſ.

$\alpha$ axis $= \frac{(\overline{m-nm}+1)n^2 + m^2}{n^2}x$, quæ autem in relatione ad w

ſimpliciſſime eſt $= \frac{m+n}{m}$w.

$\int\mu^2 : \int o^2 = \mu p^2 : no^2;\quad \mu\int^2 = \frac{(pm^2 + w\,\overline{m+n}^2)\,w}{4m^2};\ = \int o^2 =$

$(To - T\int)^2 = (To - \mu\int)^2 = (\text{ſec. } \beta \text{ axis} - \text{ſec. } \alpha \text{ diam.})^2$

$$= \frac{(pm^2 + w\,\overline{m+n}^2)\,n^2w}{m^4} + \frac{(2m - n)^2\,(pm^2 + w\,\overline{m+n}^2)\,w}{4m^4}$$

$$- \frac{2\sqrt{(pm^2 + w\,\overline{m+n}^2)\,n^2w} \,.\, \sqrt{\overline{(2m-n)}^2 \,.\, (pm^2 + w\,\overline{m+n}^2)\,w}}{m^2 \,.\, 2m^2}$$

$$= \frac{(4n^2 + \overline{2m-n}^2)\,(pm^2 + w\,\overline{m+n}^2)\,w - 4n\,\overline{2m-n}\,(pm^2 + w\,\overline{m+n}^2)\,w}{4m^4}$$

$$= \frac{[4n^2 + \overline{2m - 5n} \,.\, \overline{2m-n}] \,.\, (pm^2 + w\,\overline{m+n}^2)\,w}{4m^4}$$

$$= \frac{(4n^2 + 4m^2 - 12mn + 5n^2) \,.\, (pm^2 + w\,\overline{m+n}^2)\,w}{4m^4}$$

$$= \frac{\overline{2m - 3n}^2\,(pm^2 + w\,(m+n)^2)\,w}{4m^4} = \int o^2;$$ adeoque $\int o =$

$\frac{\overline{2m - 3n} \,.\, \sqrt{(pm^2 + w\,\overline{m+n}^2)\,w}}{2m^2};\quad \mu p^2 = \frac{(w + p)\,w}{4};$

Ergo $no^2 = \frac{\overline{w + pw}}{4} \,.\, \frac{\overline{2m - 3n}^2\,(pm^2 + w\,\overline{m+n}^2)\,w}{4m^4} :$

$\frac{(pm^2 + w\,.\,\overline{m+n}^2)\,w}{4m^2} = \frac{(w+p)w \,.\, (2m - 3n)^2}{4m^2}$ & $no =$



$2m -$

$\frac{\overline{2m-3n}\sqrt{\overline{w+pw}}}{2m}$; Porro eſt $an = \frac{no^2}{\pi} = \frac{\overline{w+pw}\,.\,\overline{2m-3n}^2}{4m^2\,.\,\overline{w+p}} =$

$\frac{\overline{2m-3n}^2}{4m^2}w.$

Subſ. $\beta$. diam. = ſ$a + an = \left(\frac{\overline{2m-n}}{4m} + \frac{\overline{2m-3n}^2}{4m^2}\right)w$

$= \frac{\overline{2m-n}m + \overline{2m-3n}^2}{4m^2}w = \frac{6m^2 - 13mn + 9n^2}{4m^2}w$

Porro $g$ſ $= \frac{CT}{2} = \frac{m-n}{2m}w$ vid. §. XI.

ſubſ. $\beta$ — ſubt. $\beta = \zeta =$ ſ$a - an = \left(\frac{\overline{2m-n}}{4m} - \frac{\overline{2m-3n}^2}{4m^2}\right)w$

$= \frac{\overline{2m-n}\,.\,m - \overline{2m-3n}^2}{4m^2}w = \frac{-2m^2 + 11mn - 9n^2}{4m^2}w$

ſubt. $\beta = \frac{(2m-3n)^2}{2m^2}w;$

ſubt. $\alpha = gp = \frac{AC}{2} = \frac{AP}{2} = \frac{w}{2};$

tang. $\alpha^2 = \mu g^2 = \frac{\text{tang. } \alpha^2 \text{ axis}}{4} = \frac{4w+pw}{4}; \quad \mu g = \frac{\sqrt{\overline{4w+pw}}}{2};$

tang. $\beta^2 = \frac{\overline{2n-m}^2\,.\,(pm^2+wm+n)w}{4m^4}$ & tang. $\beta =$

$\frac{\overline{2n-m}\sqrt{\overline{(pm^2+wm+n)w}}}{2m^2}$; nam $\mu p : \mu A = \mu V : \mu P$; $\mu p =$

$\mu A$

$\frac{\mu A}{2}$; ergo $PV = Em = \frac{P\mu}{2} = \frac{z}{2}$; itaque $om\left(\frac{n}{m} - \frac{1}{2}\right)z = \frac{2n - m}{2m}\sqrt{p\mathrm{w}}$; $om^2 = \frac{\overline{2n - m}^2}{4m^2} p\mathrm{w}$; Porro $y^2$ : ſec. $\beta^2$ axis = $\frac{(2n - m)^2}{4m^2}\mathrm{w}p : oſ^2$; ſive $\frac{n^2}{m^2}p\mathrm{w} : \frac{(pm^2 + \mathrm{w}\overline{m + n}^2)n^2\mathrm{w}}{m^4} = \frac{(2n - m)^2 p\mathrm{w}}{4m^2} : oS^2 = \frac{\overline{2n - m}(pm^2 + \mathrm{w}\overline{m + n}^2)\mathrm{w}}{4m^4}$.

*Tabula* IV.

Rationes functionum diametrorum in ſe ſpectatæ, & quidem ſi ordinatæ *ſupra* verticem axis cadunt.

| | | | |
|---|---|---|---|
| $\mathfrak{p}$ | : $\mathfrak{w}$ | $= \frac{(2m - 3n)^2}{m}$ | : $m$ |
| „ | : $\mathfrak{y}^2$ | $= 1$ | : $\mathrm{w} + p$ |
| $\mathfrak{z}^2$ | : $\mathfrak{y}^2$ | $= m$ | : $\frac{(m - 3n)^2}{m}$ |
| $\mathfrak{z}$ | : $\mathfrak{y}$ | $= m$ | : $2m - 3n$ |
| $\mathfrak{w}$ | : ſubſ. $\beta$ | $= m$ | : $\frac{6m^2 - 13mn + 9n^2}{m}$ |
| „ | : „ $\alpha$ | $= m$ | : $2n$ |
| „ | : ſubt. $\beta$ | $= m$ | : $\frac{2(2m - 3n)^2}{m}$ |
| „ | : „ $\alpha$ | $= 1$ | : $2$ |
| „ | : $\zeta$ | $= m$ | : $\frac{-2m^2 + 11mn - 9n^2}{m}$ |



tang.

| | | | |
|---|---|---|---|
| tang. $\alpha^2$ | : tang. $\beta^2$ | $= m$ | $: \dfrac{(2n-m)^2(pm^2+w\,\overline{m+n}^2)}{m^3(4w+p)}$ |
| „ | : ſubt. $\alpha$ | $= 4w+p$ | $: 2$ |
| „ | : „ $\beta$ | $= m$ | $: \dfrac{2(2m-3n)^2}{(4w+p)m}$ |
| tang. $\beta^2$ | : „ $\alpha$ | $= m$ | $: \dfrac{\overline{2n-m}^2(pm^2+w\,\overline{m+n}^2)}{2m^3}$ |
| „ | : ſubt. $\beta$ | $= \overline{2n-m}^2$ | $: \dfrac{\overline{2m-3n}^2\,2m^2}{pm^2+w(m+n)^2}$ |
| ſubt. $\alpha$ | : ſubt. $\beta$ | $= m$ | $: \dfrac{(2m-3n)^2}{m}$ |
| „ | : ſubſ. $\beta$ | $= m$ | $: \dfrac{6m^2-13mn+9n^2}{2m}$ |
| „ | : ſubſ. $\alpha$ | $= m$ | $: n$ |
| ſubſ. $\alpha$ | : ſubſ. $\beta$ | $= n$ | $: \dfrac{6m^2-13mn+9n^2}{2m}$ |
| „ | : tang. $\alpha^2$ | $= n$ | $: \dfrac{(4w+p)m}{2}$ |
| „ | : tang. $\beta^2$ | $= n$ | $: \dfrac{(2n-m)^2(pm^2+w\,\overline{m+n}^2)}{2m^3}$ |

Ob nimiam formularum longitudinem plures rationes ponere nolui.

Tabula

*Tabula* V.

Rationes functionum diam. ad ſubſecantes axis pro *caſu primo*, ſi ordinatæ cadunt *infra A.*

| | | | | | |
|---|---|---|---|---|---|
| Subſ. $\beta$ axis | : ſubſ. $\beta$ diam. | = | $m+n$ | : | $\frac{m}{2}$ |
| " | : " $\alpha$ | = | " | : | $\frac{(2m-n)m}{2n}$ |
| " | : ſubt. $\beta$ | = | " | : | $\frac{n}{2}$ |
| " | : " $\alpha$ | = | " | : | $\frac{(2m-n)^2}{2n}$ |
| " | : tang. $\beta^2$ | = | " | : | $\frac{(pm^2+4wn^2)n}{4m^2}$ |
| " | : " $\alpha^2$ | = | " | : | $\frac{\overline{2m-n}^2(4w+p)}{4n}$ |
| " | : ſubſ. $\beta$ — ſubt. $\beta$ | = | " | : | $\frac{m-n}{2}$ |
| " | : ſubſ. $\beta$ — $\mathfrak{p}$ | = | " | : | $\frac{(2m-n)}{4}$ |
| " | : $\mathfrak{w}$ | = | " | : | $\frac{(2m-n)^2}{4n}$ |
| " | : $\mathfrak{p}$ | = | " | : | $\frac{n}{4}$ |
| " | : $\mathfrak{z}^2$ | = | " | : | $\frac{(2m-n)^2(pm^2+wn^2)}{4n}$ |
| " | : $\mathfrak{y}^2$ | = | " | : | $\frac{(pm^2+wn^2)n}{4m^2}$ |
| Subſ. $\alpha$ axis | : ſubſ. $\beta$ | = | " | : | $\frac{n}{2}$ |



Subſ.

ſubſ. $\alpha$ axis : ſubſ. $\alpha$ diam. $= m+n : \frac{(2m-n)}{2}$

„ : ſubt. $\beta$ $=$ „ $: \frac{n^2}{2m}$

„ : „ $\alpha$ $=$ „ $: \frac{(2m-n)^2}{2m}$

„ : tang. $\beta$ $=$ „ $: \frac{(pm^2+4wn^2)n^2}{4m^3}$

„ : „ $\alpha^2$ $=$ „ $: \frac{(2m-n)^2(4w+p)}{4m}$

„ : ſubſ. $\beta$ — ſubt. $\beta$ $=$ „ $: \frac{(m-n)n}{2m}$

„ : ſubſ. $\beta - \mathfrak{p}$ $=$ „ $: \frac{(2m-n)n}{4m}$

„ : $\mathfrak{w}$ $=$ „ $: \frac{(2m-n)^2}{4m}$

„ : $\mathfrak{p}$ $=$ „ $: \frac{n^2}{4m}$

„ : $\mathfrak{z}^2$ $=$ „ $: \frac{(2m-n)^2(pm^2+n^2w)}{4m^3}$

„ : $\mathfrak{y}^2$ $=$ „ $: \frac{(pm^2+wn^2)n^2}{4m^3}$.

*Tabula* VI.

Pro *caſu ſecundo*, ſi ordinatæ cadunt *ſupra A*.

ſubſ. $\beta$ axis : ſubſ. $\beta$. diam. $= m+n : \frac{6m^2-13mn+9n^2}{4n}$

„ : „ $\alpha$ $=$ „ $: \frac{m}{2}$

ſubſ.

| | | | | | |
|---|---|---|---|---|---|
| ſubſ. $\beta$ axis | : ſubt. $\beta$ diam. | = | $m+n$ | : | $\frac{(2m-3)n^2}{2n}$ |
| „ | : „ $\alpha$ | = | „ | : | $\frac{m^2}{2n}$ |
| „ | : tang. $\beta^2$ | = | „ | : | $\frac{(2m-n)^2(pm^2+\overline{wm+n}^2)}{4nm^2}$ |
| „ | : „ $\alpha^2$ | = | „ | : | $\frac{m^2(4w+p)}{4n}$ |
| „ | : $\zeta$ | = | „ | : | $\frac{-2m^2+11mn-9n^2}{4n}$ |
| „ | : $\mathfrak{p}$ | = | „ | : | $\frac{(2m-3n)^2}{4n}$ |
| „ | : $\mathfrak{w}$ | = | „ | : | $\frac{m^2}{4n}$ |
| „ | : $\mathfrak{y}^2$ | = | „ | : | $\frac{(2m-3n)^2\,\overline{w+p}}{4n}$ |
| „ | : $\mathfrak{z}^2$ | = | „ | : | $\frac{m^2(w+p)}{4n}$ |
| ſubſ. $\alpha$ axis | : ſubſ. $\beta$ | = | „ | : | $\frac{-6m^2+13mn-9n^2}{4m}$ |
| „ | : „ $\alpha$ | = | „ | : | $\frac{n}{2}$ |
| „ | : ſubt. $\beta$ | = | „ | : | $\frac{(2m-3n)^2}{2m}$ |
| „ | : „ $\alpha$ | = | „ | : | $\frac{m}{2}$ |
| „ | : tang. $\beta^2$ | = | „ | : | $\frac{(2m-n)^2(pm^2+\overline{w.m+n}^2)}{4m^3}$ |

ſubſ.

| | | | |
|---|---|---|---|
| ſubſ. $\alpha$ axis | : tang. $\alpha^2$ diam. | $= m+n$ | : $\frac{m.(4w+p}{4}$ |
| » | : $\zeta$ | = » | : $\frac{-2m^2+11mn-9n^2}{4m}$ |
| » | : $\mathfrak{k}$ | = » | : $\frac{(2m-3n)^2}{4m}$ |
| » | : $\mathfrak{w}$ | = » | : $\frac{m}{4}$ |
| » | : $\mathfrak{y}^2$ | = » | : $\frac{(2m-3n)^2\,\overline{w+p}}{4m}$ |
| » | : $\mathfrak{z}^2$ | = | : $\frac{m(w+p)}{4}$. |

## §. XIX.

Progredimur ad determinationem ſubſecantium ellipſeos. Sit $AP = x$; $AT = w$, $AB = a$; $NT = z$; $MP = y = \frac{n}{m}z$; $ST =$ ſubſ. $\alpha$; $SP =$ ſubſ. $\beta$; $OS = \xi$; Quia $y^2 : z^2 = (a-x)x : (a-w)w$ & $(a-x)x : y^2 = a : p$; vid. DE LA CHAPELLE p. 321. Ergo $y^2 = \frac{(a-x)px}{a}$ & $z^2 = \frac{(a-w)pw}{a}$.

## §. XX.

Quia porro $PG : AG = AG : OG$ vid. CHAPELLE p. 280. Ergo $OG = \frac{AG^2}{PG}$; $AG = \frac{a}{2}$ & $PG = \frac{a-2x}{2}$; adeoque $OG = \frac{a^2}{4} : \frac{a-2x}{2} = \frac{a}{2(a-2x)}$.

## §. XXI.

Subtangens $OP$ itaque $= OG - PG = \frac{a^2}{2(a-2x)} - \frac{a-2x}{2}$ =

$$= \frac{a^2-(a-2x)(a-2x)}{2(a-2x)} = \frac{a^2-(a^2-4ax+4x^2)}{2(a-2x)} = \frac{4ax-4x^2}{2(a-2x)}$$

$$= \frac{(a-x)\,2x}{a-2x}.$$

§. XXII.

Itaque $OA$ = ſubtangens $- x = \frac{(a-x)\,2x}{a-2x} - x =$ $\frac{2ax-2x^2-ax+2x^2}{a-2x} = \frac{ax}{a-2x}$.

§. XXIII.

Ob ſimilitudinem triangulorum $SPM$ & $STN$ eſt $y : z =$ ſubſ. $\beta$ : ſubſ. $\alpha$; $y$ autem $= \frac{n}{m}z$; adeoque ſubſ. $\beta = \frac{n}{m}$ ſubſ. $\alpha$ & quia $y^2 : z^2 = (a-x)x : (a-w)w$; erit etiam ſubſ. $\alpha^2$ : $\frac{n^2}{m^2}$ ſubſ. $\alpha^2 = (a-w)w : a - x)x$, adeoque $(a-x)x =$ $\frac{(a-w)wn^2}{m^2}$ ſive $ax - x^2 = \frac{(aw-w^2)n^2}{m^2}$ ſive $x^2 - ax = -$ $\frac{(aw-w^2)n^2}{m^2}$; $x^2 - ax + \frac{1}{4}a^2 = \frac{1}{4}a^2 - \frac{(aw-w^2)n^2}{m^2}$; adeoque radice extracta: $x - \frac{a}{2} = -\sqrt{\left(\frac{1}{4}a^2 - \frac{(aw-w^2)n^2}{m^2}\right)}$

$$x = \frac{a}{2} - \frac{\sqrt{(a^2m^2-(aw-w^2)\,4n^2)}}{2m} = \frac{am-\sqrt{[a^2m^2-(a-w)4wn^2]}}{2m}.$$

§. XXIV.

Determinata abſciſſa $x$, ſubtangens $OP = \frac{(a-x)\,2x}{a-2x}$ in alia expreſſione prodit $= \left[a - \frac{(am-\sqrt{(a^2m^2-(a-w)\,4wn^2)}}{2m}\right]$



(am

$$\frac{(am - \surd(a^2m^2 - \ldots)}{m} : a - \frac{(am - \surd(a^2m^2 - \ldots)}{m} =$$

$$\frac{am + \surd(a^2m^2\ldots)(am - \surd a^2\ldots)}{2m\surd(a^2\ldots)} = \frac{a^2m^2 - (a^2m^2 - (aw - w^2)4n^2)}{2m\surd(a^2m^2\ldots)}$$

$$= \frac{(a-w)\,2wn^2}{m\surd a^2\ldots}.$$

## §. XXV.

$$OT = \text{ſubtang.}\,\beta + w - x = \frac{(a-w)\,2wn^2}{m\surd a^2\ldots} + w -$$

$$\frac{(am - \surd a^2\ldots)}{2m} = \frac{(a-w)4wn^2 + (2mw - am)\surd(a^2\ldots) + a^2m^2 - (a-w)4wn^2}{2m\surd a^2m^2\ldots}$$

$$= \frac{a^2m^2 + (2w-a)m\surd a^2\ldots}{2m\surd(a^2\ldots)} = \frac{a^2m + (2w-a)\surd a^2\ldots}{2\surd a^2\ldots}.$$

## §. XXVI.

Hinc ſubſ. $\alpha = OT + \xi = \dfrac{a^2m + (2w-a)\surd a^2\ldots}{2\surd a^2\ldots} + \xi$; A)

$\dfrac{n}{m}$ ſubſ. $\alpha$ = ſubſ. $\beta = OP + \xi = \dfrac{(a-w)2wn^2}{m\surd a^2\ldots} + \xi$; $\dfrac{\text{ſubſ.}\,\alpha}{m} = \dfrac{(a-w)2wn}{m\surd a^2\ldots} + \dfrac{\xi}{n}$ ſubſ. $\alpha = \dfrac{(a-w)2wn}{\surd a^2\ldots} + \dfrac{m}{n}\xi$; B) quia æquatio $A = B$; erit etiam $\dfrac{(a-w)\,2wn}{\surd a^2\ldots} + \dfrac{m}{n}\xi = \dfrac{a^2m + (2w-a)\surd a^2\ldots}{2\surd a^2\ldots}$ $+\xi$; $\dfrac{m-n}{n}\xi = \dfrac{a^2m + (2w-a)\surd(a^2\ldots) - (a-w)4wn}{2\surd a^2\ldots}$;

$$\xi = \frac{[a^2m - (a-w)4wn + (2w-a)\surd a^2\ldots]n}{(m-n)2\surd a^2\ldots}.$$

## §. XXVII.

Determinato $\xi$ ſubſecans $\beta = PS$ erit $=$ ſubtang. $OP + \xi$

$$= \frac{(a-w)2wn^2}{m\sqrt{a^2\ldots}} + \frac{[a^2m - (a-w)4wn + (2w-a)\sqrt{a^2\ldots}]n}{\overline{m-n}\,2\sqrt{a^2\ldots}} =$$

$$\frac{\overline{a-w}\,4wn^2\,.\,\overline{m-n} + [(a^2m - (a-w)4wn + (2w-a)\sqrt{a^2\ldots})]\,.\,mn}{\overline{m-n}\,2m\sqrt{a^2\ldots}} =$$

$$\frac{\overline{m-n} - m\,.\,\overline{a-w}\,.\,4wn^2 + a^2m^2n + \overline{2w-a}\,.\,mn\sqrt{a^2\ldots}}{\overline{m-n}\,2m\sqrt{a^2\ldots}} =$$

$$\frac{[(a^2m + \overline{2w-a}\sqrt{a^2\ldots})m - (a-w)4wn^2]n}{\overline{m-n}\,2m\sqrt{a^2\ldots}}.$$

## §. XXVIII.

Subſ. $\alpha =$ ſubſ. $\beta + w - x$

$$= \frac{[(a^2m + \overline{2w-a}\sqrt{a^2\ldots})m - \overline{a-w}\,4wn^2]n}{\overline{m-n}\,2m\sqrt{a^2\ldots}} + \frac{\overline{2w-a}\,.\,m + \sqrt{a^2\ldots}}{2m}$$

$$= \frac{\text{ſubſ.}\,\beta + \overline{2w-am}\,.\,\overline{m-n}\sqrt{a^2\ldots}) + (a^2m^2 - (a-w)4wn^2)\overline{m-n}}{\overline{m-n}\,2m\sqrt{a^2\ldots}}$$

$$= \frac{\overline{n+m-n}\,\overline{2w-a}\,m\sqrt{a^2\ldots}) + \overline{n+m-n}\,a^2m^2 + (n-m+n)\overline{a-w}\,4wn^2}{\overline{m-n}\,2m\sqrt{a^2\ldots}}$$

$$= \frac{\overline{2w-a}\,m\sqrt{a^2\ldots}) + a^2m^2 - (a-w)4wn^2}{\overline{m-n}\,2\sqrt{a^2\ldots}}.$$

## §. XXIX.

Spatium ſuadet, ſubſiſtere in paucis hisce formulis pro ſubſecantibus Ellipſeos ad nauſeam usque extenſis, nec plures, easque longiores, quærere. Reliquum eſſet, ut ſubſecantium Hyperbolæ formulas adhuc inveniremus, ſed, credo, abſoluturum me ab opera quemlibet eſſe, qui non ignorat, functiones Hyperbolæ ſignis modo differre a functionibus Ellipſeos.

## §. XXX.

De uſu ſubſecantium haud ægre quædam ſuperadderem, niſi temporis anguſtia me ita conſtituiſſet, ut de applicatione notatu digni nihil adjicere potuiſſem. Quod ſectiones corporum §. I. memor. attinet, ſubſecantium adjumento adhuc nil novi reperi. Pedem itaque hic figere me oportet; de applicatione novæ hujus theſeos alio forſitan tempore dicturum.

## THESES

### I.

Formula $\frac{r \cdot n}{2}$ hactenus pro dimenſione polygoni infinite parvorum laterum, circuli inſcripti, uſitata (ſcilicet *r* ſignificet radium circuli & *n* numerum infinite parvorum laterum) non exhibet quantitatem areæ polygoni *n* laterum, ſed polygoni $2n$ laterum.

### II.

Nam radius circuli eſt altitudo triangulorum polygoni circulo inſcripti *ſecundum rigorem geometricum talis*.

### III.

Orbitæ planetarum & cometarum ſecundum ſyſtema LAMBERTI non ſunt ellipſes, ſed lineæ cycloidales.

### IV.

Datur ſeries nec convergens nec divergens, quam nuncupem linearem: ſcilicet $\frac{1}{x+x} = \frac{1}{x} - \frac{1}{x} + \frac{1}{x} - \frac{1}{x} + \ldots$ ſeriem nuncupo linearem.

### V.

Ellipſis eſt ſectio

1. Coni
   a) in infinito convergentis, ſive Cylindri,
   b) in finito convergentis, ſive regularis.

2. Conoidis

2. Conoidis
   a) parabolicæ,
   b) hyperbolicæ.
3. Spæroidis ellipticæ in quovis ejus puncto.

VI.

Parallelogrammi Newtoniani ad axem in peritrochio facilis eſt applicatio.

VII.

Non datur numerus minor, quam $x^0$, quicunque etiam pro $x$ ponatur numerus.

VIII.

Axis in globo, circa ſe moto, quieſcit.

IX.

Nec numeri, nec lineæ, nec plana, nec ſolida concipi poſſunt infinite magna.

X.

Corpora ſub antlia pnevmatica in Luna tardius humi cadere debent, quam in Terra.

XI.

Corpora, quæ in ſuperficiem Terræ verticaliter cadere nituntur, ſæpius deſcribunt lineam curvam, quod etiam accideret, ſi atmoſphæra non eſſet.

XII.

Iterum, etiamſi atmoſphæra mox denſior, mox rarior non eſſet, nihilominus quotidie modo tardius modo celerius corpora ſuperficiem terræ cadendo tangerent.

XIII.

Solem nos nunquam, & ſtellas quasdam tantum nonunquam in veris ſuis locis obſervamus.

XIV.

Tot dantur eodem temqore irides, quot ſunt obſervatores.

XV.

Siquis agrum, figuram rhomboidis habentem, cujus primum latus $\frac{5}{2}$ decempedas, ſecundum etiam $\frac{5}{2}$ dec., tertium 4 dec., quartum totidem dec., commutat cum alio, ejusdem figuræ, cujus primum latus $\frac{5}{2}$ dec.,

dec., ſecundum $\frac{1}{2}$ dec. tertium 3 dec. & quartum etiam 3 dec.; commutatio erit juſta.

XVI.

Dimidia fere pars incolarum Lunæ inſigni caret ſpectaculo, Terræ ſcilicet diſcum per ætatem nunquam videns.

---

*Animadvertenda.*

Vbi expreſſiones $\surd\,(\ldots)w$ occurrunt, $w$ &, ſi habet, ejus factorem, etiam ſub ſigno $\surd$ poſitum intelligo. pag. 6. lin. penult. leg. §. III. pag. 11. §. VIII. leg. $= y : z$; pag. 18. lin. 4. leg. $pV$; ibid. lin. antepenult. leg. ord. $\mu P$; & ult. leg. cum cum curva in $\mu$ vergit; pag. 20. l. 4. leg. $= \pi : \frac{1}{w}$; ib. lin. 9. leg. $fb = 2w$; ib. lin. penult. leg. $\frac{(4n^4 w}{m^4} +$.

PRAECLARISSIMO ATQVE DOCTISSIMO
DOMINO CANDIDATO AVCTORI
PRÆSES
*S. P. D.*

*Cum mihi diſſertationem Tuam offerres, ratio valetudinis meæ non permiſit, ut attente illam perlegerem; hinc TIBI, auditori meo in diſciplinis, quas trado, ſemper ſolertiſſimo, nullam remoram objicere volui, quo minus eam publici juris faceres, qualem elaboraſti, fretus Tuæ ſcientiæ in hoc ſtudiorum genere, & hoc ſpecimen abunde declarat, TE in* analyſi *magnos profectus feciſſe: ſolviſti non* ſecuiſti *nodum hujus quæſtionis. Ut tangas, non* ſeces *veritatem, ex animo TIBI apprecor, & fauſtos in omnibus Tuis rebus, quibus dignus es, ſucceſſus, ut habeas, votorum meorum & bonorum omnium ſumma eſt.* Tubingæ 25 *Aug.* 1779.

## Ad §§. III & ſeqq.

Neſcio, quomodo evenit, ut æquationem $ſ^2 = 2ſx + dx$; §. III. memor. non attentius conſiderarem; cum enim ultimæ jam paginæ prelum relinquerent, demum mihi in mentem venit, valorem pro $ſ$ etiam ex hac æquatione facile poſſe erui: nam $ſ^2 - 2ſx = dx$; $ſ^2 - 2ſx + x^2 = x^2 + dx$; $ſ - x = \surd[(x+d)x]$; $ſ = x + \surd[(x + w - x)x]$; $ſ = x + \surd(wx)$; atque ſic gaudeo formulam generaliorem pro ſubſecanti $TP$ opportuno adhuc tempore eruiſſe, quoniam altera magis habet formam arithmeticam, quæ prodit, ſi pro $x$ ſubſtituitur valor $\frac{n^2}{m^2} w$;

ſubſ. $\alpha$ itaque $= x + \surd(wx) + w - x = w + \surd(wx)$

$\xi = x + \surd(wx) - 2x = \surd(wx) - x$

$AT = \surd(wx)$; $FT = w - \surd(wx)$;

ſec. $\alpha = \surd([w + x + p + 2\surd(wx)]w)$

ſec. $\beta = \surd([w + x + p + 2\surd(wx)]x)$.

Niſi temporis anguſtia vetaret, plura lubenter adderem.

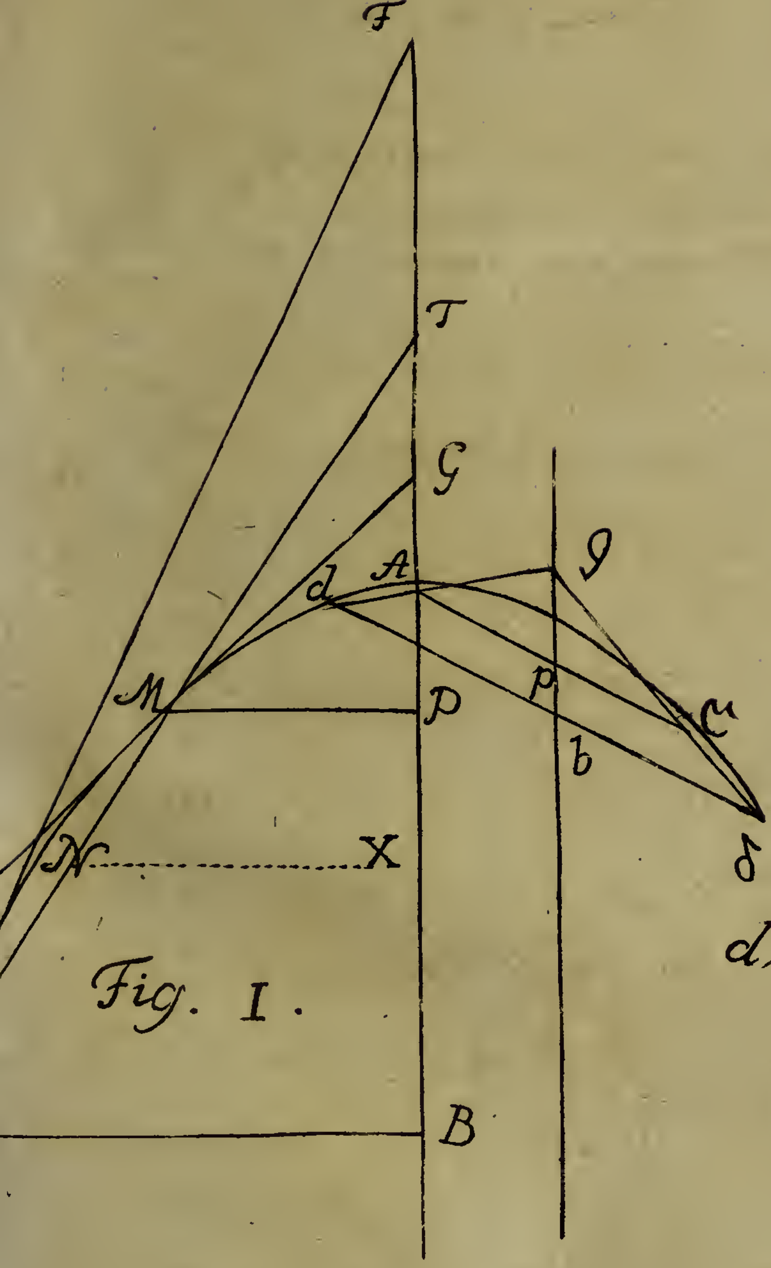


Fig. I.

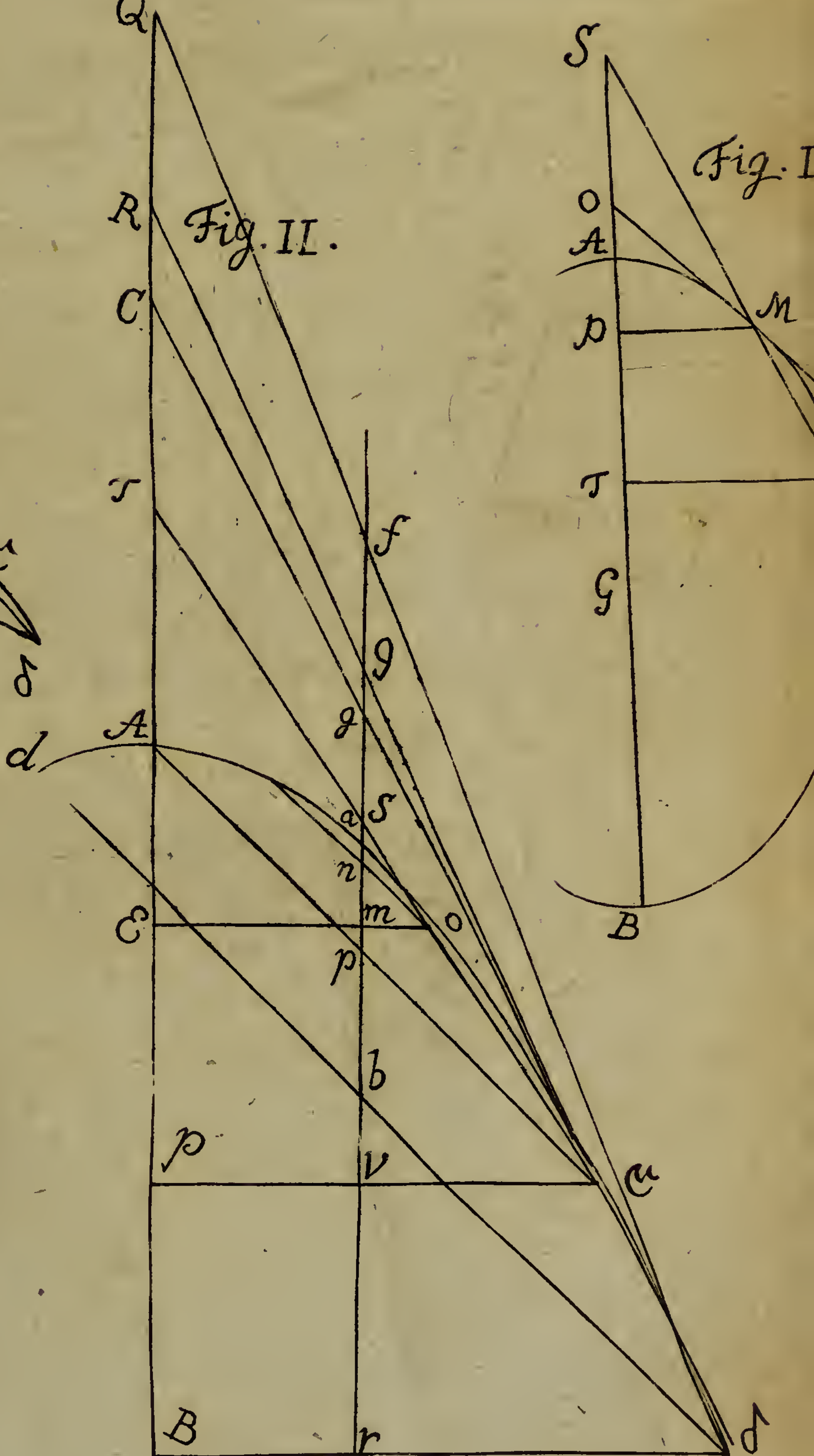


Fig. II.

Fig. I